Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Thursday 25 May 2023

Afternoon

Paper reference **8MA0/22**

Mathematics

Advanced Subsidiary
PAPER 22: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, wherever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 30. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1.

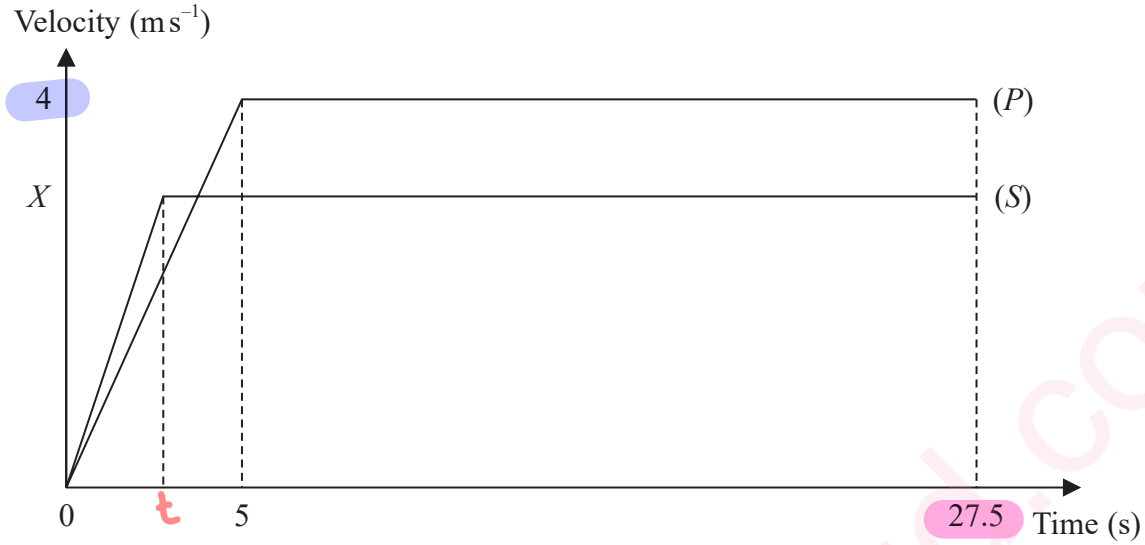


Figure 1

22.5

Two children, Pat (P) and Sam (S), run a race along a straight horizontal track.

Both children start from rest at the same time and cross the finish line at the same time.

In a model of the motion:

Pat accelerates at a constant rate from rest for 5 s until reaching a speed of 4 m s^{-1} and then maintains a constant speed of 4 m s^{-1} until crossing the finish line.

Sam accelerates at a constant rate of 1 m s^{-2} from rest until reaching a speed of X m s^{-1} and then maintains a constant speed of X m s^{-1} until crossing the finish line.

Both children take 27.5 s to complete the race.

The velocity-time graphs shown in Figure 1 describe the model of the motion of each child from the instant they start to the instant they cross the finish line together.

Using the model,

(a) explain why the areas under the two graphs are equal,

(1)

(b) find the acceleration of Pat during the first 5 seconds,

(1)

(c) find, in metres, the length of the race,

(2)

(d) find the value of X, giving your answer to 3 significant figures.

(4)

@ Because the distance travelled by both children are equal

Question 1 continued

(b) $a = \frac{v - u}{t}$

$u = 0$
$v = 4$
$t = 5$

$a = \frac{4 - 0}{5} = 0.8 \text{ m/s}^2$

(c) race length = distance travelled = area under the v-t graph

area under the P graph:

using area of trapezium = $\frac{1}{2}(a + b) \times h$

distance = $\frac{1}{2} \times (27.5 + 22.5) \times 4$ = 100m

(d) area under graph S = 100m

to find t:
using $t = \frac{v - u}{a}$

$v = X$
$u = 0$
$a = 1$

$t = \frac{X - 0}{1}$
$\therefore t = X$

Question 1 continued

area under S:

Using area of trapezium $= \frac{1}{2}(a+b) \times h$

$100\text{m} = \frac{1}{2}(27.5 + 27.5 - X) \times X$

$100 = 27.5X - \frac{1}{2}X^2$

$X^2 - 55X + 200 = 0$

$X = \frac{55 \pm \sqrt{55^2 - 4(200)}}{2}$

$X = 3.915...$ or $-3.423...$

We know $X > 0$, because Sam is moving forward.

$\therefore X = 3.92$ (to 3sf)

2. A small stone is projected vertically upwards with speed $39.2\,\text{m s}^{-1}$ from a point O.

The stone is modelled as a particle moving freely under gravity from when it is projected until it hits the ground 10 s later.

Using the model, find

(a) the height of O above the ground,

(3)

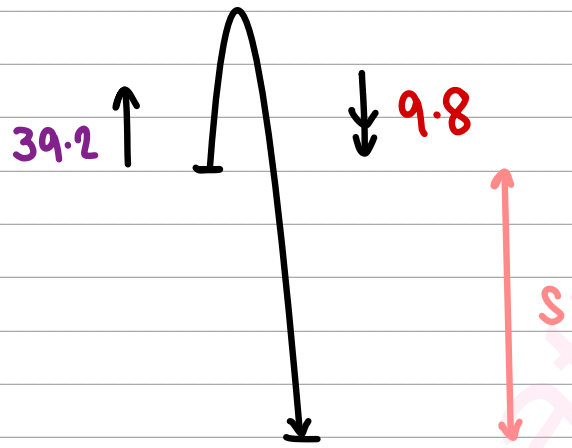
(b) the total length of time for which the speed of the stone is less than or equal to $24.5\,\text{m s}^{-1}$

(3)

(c) State one refinement that could be made to the model that would make your answer to part (a) more accurate.

(1)

(a)



using $s = ut + \frac{1}{2}at^2$

treating upwards (↑) as positive

$u = 39{\cdot}2$
$t = 10$
$a = -9{\cdot}8$
$s = -s$

$-s = 39{\cdot}2(10) + (\frac{1}{2} \times -9{\cdot}8 \times 10^2)$

$-s = 392 - 490 = \boxed{-98\,\text{m}}$

Question 2 continued

(b) find the times when speed = 24·5

treating upwards (↑) as positive

using $t = \frac{v - u}{a}$

$u = 39{\cdot}2$

$t = t$

$v = 24{\cdot}5$ OR $-24{\cdot}5$

$a = -9{\cdot}8$

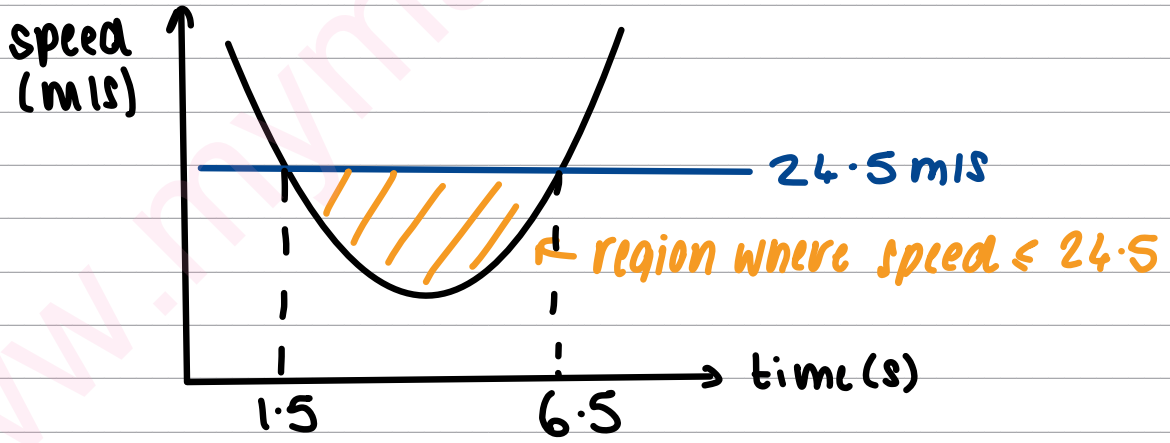
case 1: $t = \frac{24{\cdot}5 - 39{\cdot}2}{-9{\cdot}8}$

$t = 1{\cdot}5\text{s}$

case 2: $t = \frac{-24{\cdot}5 - 39{\cdot}2}{-9{\cdot}8}$

$t = 6{\cdot}5\text{s}$

speed time graph for the stone:



∴ time where speed ≤ 24·5 m/s = 6·5 − 1·5 = 5 seconds

= 5 seconds

(c) include air resistance

(Total for Question 2 is 7 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A fixed point O lies on a straight line.

A particle P moves along the straight line such that at time t seconds, $t \geqslant 0$, after passing through O, the velocity of P, $v\,\text{m s}^{-1}$, is modelled as

$$v = 15 - t^2 - 2t$$

(a) Verify that P comes to instantaneous rest when $t = 3$

(1)

(b) Find the magnitude of the acceleration of P when $t = 3$

(3)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(4)

(a) When $t=3$: $v = 15 - (3)^2 - 2(3)$

$v = 15 - 9 - 6$

$v = 0$ m/s

∴ P comes to an instantaneous rest when $t = 3$s

(b) $a = \frac{dv}{dt}$

$a = -2t - 2$

When $t=3$, $a = -2(3) - 2$

$a = -8$ m/s^2

∴ the magnitude of a = 8 m/s^2

(c) First find the roots of $v = 15 - t^2 - 2t$

$v = -(t^2 + 2t - 15)$

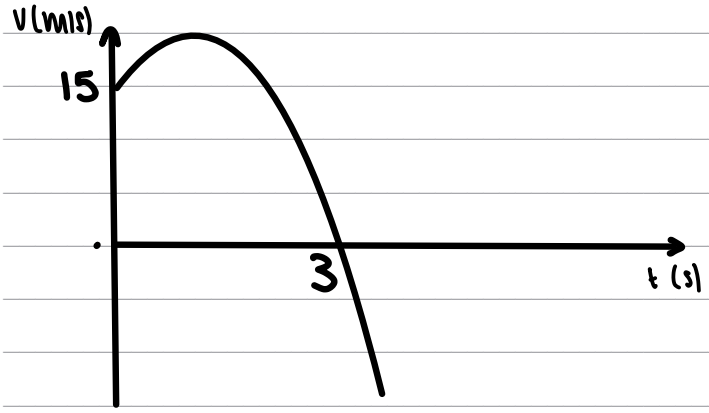
$v = -(t+5)(t-3)$

$v = 3$ or -5

↑ discard as $t \geqslant 0$

Question 3 continued

Sketch the graph for $v = 15 - t^2 - 2t$



We can see that when $t = 3$, $v = 0$

∴ when $t > 3$, P changes direction as its velocity is negative

So we must split the integral up to find distance:

$$\text{distance} = \int_0^3 (15 - t^2 - 2t)\,dt + \int_4^3 (15 - t^2 - 2t)\,dt$$

$$= \left[15t - \frac{1}{3}t^3 - t^2\right]_0^3 + \left[15t - \frac{1}{3}t^3 - t^2\right]_4^3$$

$$= \left[15(3) - \frac{1}{3}(3)^3 - (3)^2\right] - 0 + \left[15(3) - \frac{1}{3}(3)^3 - 3^2\right] - \left[15(4) - \frac{1}{3}(4)^3 - 4^2\right]$$

$$= \frac{94}{3}\text{ m}$$

(Total for Question 3 is 8 marks)

4.

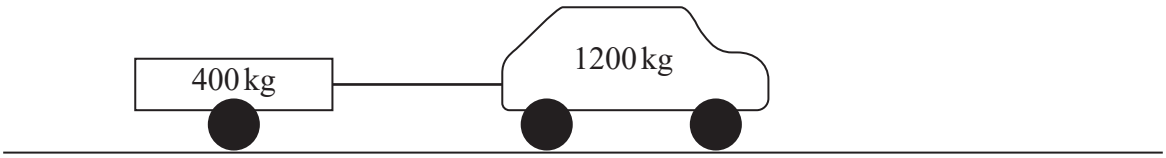


Figure 2

A car of mass 1200 kg is towing a trailer of mass 400 kg along a straight horizontal road using a tow rope, as shown in Figure 2.
The rope is horizontal and parallel to the direction of motion of the car.

- The resistance to motion of the car is modelled as a constant force of magnitude $2R$ newtons
- The resistance to motion of the trailer is modelled as a constant force of magnitude R newtons
- The rope is modelled as being light and inextensible
- The acceleration of the car is modelled as $a\,\text{m s}^{-2}$

The driving force of the engine of the car is 7400 N and the tension in the tow rope is 2400 N.

Using the model,
(a) find the value of a

(5)

In a refined model, the rope is modelled as having mass and the acceleration of the car is found to be $a_1\,\text{m s}^{-2}$

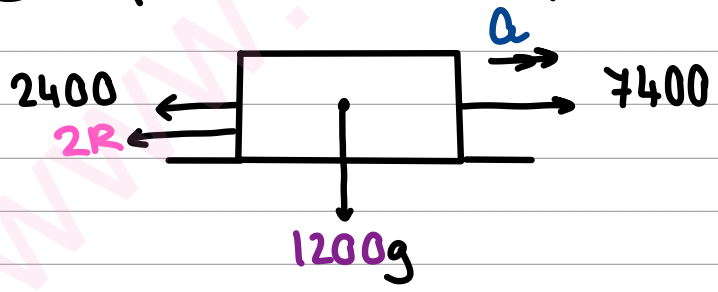
(b) State how the value of a_1 compares with the value of a

(1)

(c) State one limitation of the model used for the resistance to motion of the car.

(1)

(a) Equation of motion for the car :



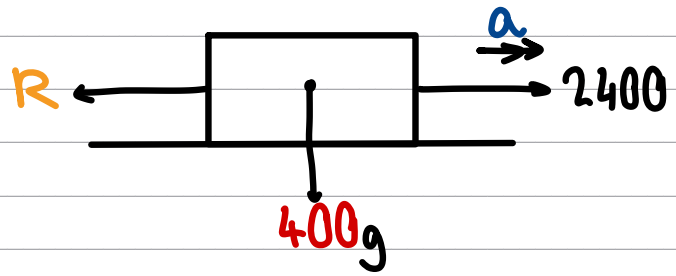
Using F = ma :

R(→): $7400 - 2400 - 2R = 1200a$

$5000 - 2R = 1200a$

Question 4 continued

Equation of motion for the trailer:



$R(\rightarrow)$: $2400 - R = 400a$

now finding a:

$R = 2500 - 600a$

$R = 2400 - 400a$

$2400 - 400a = 2500 - 600a$

$200a = 100$

$a = 0.5 m/s^2$

(b) The value of a, would be less than a

(c) The resistance will not be constant.